SEMESTRAL EXAMINATION
B. MATH II YEAR, I SEMESTER 2010-2011
ANALYSIS III

Max. 100. Time limit: 3hrs

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1. Find all intervals (finite/infinite, open/closed/half-open) on which the series $\sum_{n=1}^{\infty} \frac{e^{nx}}{n!}$ is uniformly convergent. Justify your answer. [15]

2. Prove or disprove: functions of the type $a_0 + a_1(\sin^2 x) + a_2(\sin^2 x)^2 + \ldots + a_n(\sin^2 x)^n$ $(n \geq 1, a_i's \in \mathbb{R})$ form a dense subset of $C[0,1]$.

($C[0,1]$ is the set of all continuous functions : $[0,1] \to \mathbb{R}$ with the metric d defined by $d(f,g) = \sup\{|f(x) - g(x)| : 0 \leq x \leq 1\}$]. [15]

3. Let $u(x,y) = \frac{x+1}{(x+1)^2+y^2}$. Does there exist a function $v(x,y)$ such that $udx + vdy$ is exact? If so, find one such v. [10]

4. Define $d(x,y) = \begin{cases} 0 \text{ if } x = y \\ \|x\| + \|y\| \text{ if x} \neq y \end{cases}$ for all $x, y \in \mathbb{R}^n$. Here $\|x\|$ is the usual Euclidean norm on $\mathbb{R}^n$. Prove that a set A is open in $(\mathbb{R}^n, d)$ if and only if either $0 \notin A$ or $0 \in A$ and A contains $(-\delta, \delta) \times \ldots \times (-\delta, \delta)$ for δ sufficiently small. [15]

5. Let $f(x,y) = (2xe^y + y, x^2e^y + x - 2y)$. Does there exist a differentiable function $\phi : \mathbb{R}^2 \to \mathbb{R}$ such that $f = \nabla\phi$?. If so, find one such function. [10]

6. Evaluate the surface integral of the function $F(x,y,z) = (xy, yz, x+z)$ over the surface $x^2 + y^2 + z^2 = 6, 0 \leq z \leq \sqrt{6}$ using DivergenceTheorem. [15]

7. Evaluate $\int_\Gamma (1 + 2xy + y^2)dx + (-2xy - x^2)dy$ where Γ is the rectangle $[-1,1] \times [2,3]$ using Green's Theorem. [5]

8. Find the volume of the solid inside the cylinder $x^2 + y^2 - 4y = 0$ lying between the plane $z = 0$ and the surface $x^2 + y^2 = 8z$. [15]